SS – 349

V Semester B.A./B.Sc. Examination, November/December 2018
(Semester Scheme) (Repeaters – Prior to 2016 – 17)
(NS – 2013 – 14 and Onwards)
MATHEMATICS – V

Time : 3 Hours　　　　　　　　　　　　　　　　　　Max. Marks : 100

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions.　　　　　　　　　　　　　(15×2=30)

1) In a vector space V(F) show that $C.0 = 0 \ \forall \ C \in F$.

2) Verify the subset $W = \{(x_1, x_2, x_3)/x_1^2 + x_2^2 + x_3^2 \le 0\}$ of $V_3(R)$ is a subspace of $V_3(R)$.

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3) Verify the set S = {(1, 0, 1), (0, 2, 2), (3, 7, 1)} is linearly independent.

4) Define linear transformation of a vector space.

5) Find the matrix of linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (2x + 3y, 4x - 5y)$ w.r.t. standard basis.

6) Find k, so that (1, k, 5) is a linear combination of (1,–3, 2) and (2, –1, 1).

7) Find the unit tangent 't' for the space curves $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at t = 1

8) For a space curve define curvature and torsion at any point on the curve.

9) If $\vec{r} = t\hat{i} + t^2\hat{j}$ find $\frac{d\vec{r}}{dt}$ at t = 1.

10) Find the normal vector to the cylinder $(x - 1)^2 + (y + 2)^2 = 4$ at $(2, \sqrt{3} - 2, 0)$.

11) Find the tangent plane to the surface $x^2 + y^2 = 4$ at $(1, \sqrt{3}, 2)$.

12) If $\phi(x, y, z) = 3x^2y - y^3z^2$ find $\nabla\phi$ at (1, –2, –1).

13) Show that $\vec{F} = (6xy + z^3)\,\hat{i} + (3x^2 - z)\,\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.

14) If $\vec{F} = x^2\hat{i} + 3y\hat{j} + x^3\hat{k}$ find $\operatorname{div}\vec{F}$.

15) Show that the vector

$\vec{F} = (x + 3y)\,\hat{i} + (y - 3z)\,\hat{j} + (x - 2z)\,\hat{k}$ is solenoidal.

P.T.O.

16) If $\phi = x^2 - y^2 + 4z$, show that $\nabla^2 \phi = 0$.

17) Define complex Fourier transform of f(x).

18) Find the Fourier transform of $f(x) = e^{-ax}$, (a > 0).

19) Find the Fourier sine transform of

$$f(x) = \begin{cases} 1 & 0 \le x < a \\ 0 & x \ge a \end{cases}.$$

20) If 'a' and 'b' are constants, then for any two functions f(x) and g(x) show that $F_s\{a\,f(x) + b\,g(x)\} = a\,F_s\{f(x)\} + b\,F_s\{g(x)\}$.

II. Answer **any four** of the following. **(4×5=20)**

1) Prove that intersection of any two subspaces of a vector space V over a field F, is also a subspace of V.

2) Express the vector (3, 5, 2) as a linear combination of the vectors (1, 1, 0), (2, 3, 0), (0, 0, 1) of $V_3(R)$.

3) Prove that in an n-dimensional vector space V(F) any set of n linearly independent vectors is a basis.

4) Find the linear transformation $T : V_2(R) \to V_2(R)$ such that T (1, 2) = (3, 0) and T (2, 1) = (1, 2).

5) If $T : R^3 \to R^3$ defined by T (1, 0, 0) = (1, 1, 2), T (0, 1, 0) = (1, –1, 0), T (0, 0, 1) = (0, 0, 1). Find the range space, nullspace, rank and nullity of T and also verify the rank-nullity theorem.

6) Find the matrix of the linear transformation $T : R^2 \to R^3$ defined by T(x, y) = (x – y, x, 3x + y) with respect to a standard basis.

III. Answer **any four** of the following. **(4×5=20)**

1) State and prove Serret-Frenet formulae for a space curve.

2) Find the curvature and Torsion for the curve $x = t,\ y = t^2,\ z = \frac{2}{3}t^2$ at t = 1.

3) Find the equation of the tangent plane and normal line at (4, 0, 3) to the surface $x^2 + y^2 + z^2 - 25 = 0$.

4) Find the equation of the tangent plane and unit normal vector to the cylinder $(x-1)^2 + (y+1)^2 = 9$ at the point $(3, \sqrt{5} - 1, 0)$.

5) Show that the surface $5x^2 - 2yz - 9x = 0$ is orthogonal to the surface $4x^2y + z^3 = 4$ at (1, –1, 2).

6) The spherical co-ordinates of a point are $\left(1, \frac{\pi}{4}, 5\frac{\pi}{6}\right)$, find its Cartesian coordinates.

IV. Answer **any three** of the following. **(3×5=15)**

1) If $\vec{a} = 2x^2\hat{i} - 3yz\hat{j} + xz^2\hat{k}$ and $\phi = 2z - x^3y$ find $\vec{a} \times \nabla\phi$ at (1, –1, 1).

2) Find the directional derivative of the function $\phi(x, y, z) = xyz - xy^2z^3$ at (1, 2, –1) in the direction of the vector $\hat{i} - \hat{j} - 3\hat{k}$.

3) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

4) If $\vec{f} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and $\vec{g} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ show that $\vec{f} \times \vec{g}$ is a solenoidal.

5) Prove that $\text{curl}\,(\phi\vec{f}) = \phi\,\text{curl}\,\vec{f} + \text{grad}\,\phi \times \vec{f}$.

V. Answer **any three** of the following. **(3×5=15)**

1) Express the Fourier expansion of the function

$$f(x) = \begin{cases} \pi/2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}.$$

2) Find the Fourier transform of $f(x) = \begin{cases} 1 & 0 \le x < a \\ 0 & x \ge a \end{cases}$.

3) Find the Fourier transform of $f(x) = e^{-|x|}$.

4) Find the Fourier cosine transform of the function $f(x) = \begin{cases} x & 0 < x < \pi/2 \\ 0 & x > \pi/2 \end{cases}$.

5) Prove that $F_c[f''(x)] = -\sqrt{\frac{2}{\pi}}\, f'(0) - \alpha^2 F_c[f(x)]$.
